

## I. Kinematics: The Study of Motion

### Concept Review

#### Conflicting Contentions

1. Ball Toss
2. Galileo's Ramp
3. High Road, Low Road

#### Qualitative Reasoning Questions

1. Constant Acceleration?
2. Velocity Graph
3. Curved Ramp
4. Brothers' Race - Part 1
5. Brothers' Race - Part 2
6. Motorcycle Cop

#### Can You Explain This?

1. Weight Drop
2. Rounding A Curve
3. Walking Around A Table
4. The Velocity of a Toy Car
5. Galileo's Ramp – Revisited
6. Kinematics Graphs

## CONCEPT REVIEW

Kinematics is the study of how things move. Understanding the following definitions and principles will be both helpful and necessary for successfully answering the exercises in this section. This concept review is not meant to replace the full explanation of a textbook or lecture. If any of these principles are unfamiliar or confusing, please consult with an instructor or textbook before continuing with the warm-up exercises.

### 1. Definitions

- a. Distance is a measure of how far an object has traveled. Displacement is the measure of the position of an object with respect to an origin. Distance and displacement are different in that direction is included in the description of a displacement.
- b. Speed is a measure how fast an object is moving. Velocity is a measure of how fast an object is moving but also includes the direction of motion.
- c. Acceleration is the measure of the change in velocity of an object, and includes both magnitude and direction.

### 2. Equations

Let  $d$  be the symbol for displacement,  $t$  be the symbol for time,  $v$  be the symbol for velocity, and  $a$  be the symbol for acceleration. The symbol  $\Delta$  (the Greek capital letter, Delta) represents the change in a quantity. A change is always defined as the final value minus the initial value. Then

- a.  $\bar{v} = \Delta d / \Delta t$  : velocity is the change in displacement divided by a change in time.
- b.  $a = \Delta v / \Delta t$ : acceleration is the change in velocity divided by a change in time.
- c. If the velocity is constant, then the first equation can be manipulated to read  $d = d_0 + \bar{v}t$  where  $d$  is the final displacement,  $d_0$  is the initial position of the object at time  $t = 0$ ,  $t$  is the elapsed time and  $v$  is the velocity.
- d. If acceleration is constant, then  $d = d_0 + v_0t + 1/2at^2$  where  $d$  is the final displacement,  $d_0$  and  $v_0$  are the initial position and velocity of the object at time  $t = 0$ ,  $t$  is the elapsed time and  $a$  is the acceleration.
- e. For constant acceleration and an object starting from rest, the previous equation can be simplified to

$$d = d_0 + 1/2at^2$$

### 3. Scalars, vectors, and direction

- a. Displacement, velocity, and acceleration are examples of vectors. Vectors are quantities that are described by both a magnitude and direction. For one-dimensional motion, the direction is often represented by a plus or minus sign.
- b. The direction of a velocity vector is the direction of the change in displacement. The direction of an acceleration vector is the direction of the change in velocity.
- c. Vectors do not add and subtract like normal algebraic numbers. Instead, the correct solution, which must include both a magnitude (a number) and a direction (often an angle) can be obtained by drawing the vectors graphically or using trigonometry.

### 4. Direction in one dimension.

- a. A positive displacement is a displacement in the positive X direction; a negative displacement is a displacement in the negative X direction.
- b. A positive velocity is motion away from negative X values and toward positive X values; a negative velocity is motion away from positive X values and toward negative X values.
- c. In physics, the term 'deceleration' is not needed and serves no useful purpose. A positive acceleration is *either* an object moving in the positive direction and speeding up or moving in the negative X direction and slowing down. A negative acceleration is *either* an object moving in the negative direction and speeding up *or* moving in the positive direction and slowing down.

### 5. Kinematics graphs and motion diagrams.

- a. The slope of a displacement vs. time graph represent the velocity of the object. The slope of a velocity vs. time graph represents the acceleration of an object. Since a slope can be positive, negative or zero, the slope of the graph is able to indicate both the magnitude and direction of a velocity or acceleration.
- b. An object moving with a constant velocity has zero acceleration. Displacement is proportional to the elapsed time, meaning that a graph of position versus time is a straight line. If the object is at rest, the line is horizontal. If the velocity is positive, the slope of the line is positive; if the velocity is negative, the slope of the line is negative.
- c. An object moving with constant acceleration will have a velocity that is proportional to elapsed time, which results in a graph of velocity versus time that is linear. If the object is moving with a constant velocity, the line is horizontal (acceleration is zero). If the acceleration is positive, the slope of the line is positive; if the acceleration is negative, the slope of the line is negative.
- d. The position versus time graph for constant acceleration is not linear. The shape of the graph is a curve known as a parabola. The slope of the position vs. time graph at any given point will still represent the velocity of the object at that point.

- e. The motion of an object can also be represented in the form of a picture by drawing the position of the object at equal time intervals. If the pictures are equally spaced, the object is moving with a constant velocity. If the separation between the pictures is increasing or decreasing, the object is acceleration. Two objects have the same speed if they have the same separation between two consecutive pictures.

## CONFLICTING CONTENTIONS

### 1. Ball Toss

[NOTE: This first problem of the workbook presents an example of a very complete and thorough solution to a conceptual exercise. By the end of the course, you will improve on your ability to ‘reason through’ an answer to a conceptual question in this way. But don’t be afraid to start at the beginning with simpler answers.]

#### *The Situation*

Mr. Nicholls tossed a ball up in the air. The class observed the ball rise up to a greatest height then fall back into Mr. Nicholls’ hand. He then asked the class, “What is the acceleration of the ball at the apex of the motion, which is the highest point reached by the ball?”

Alonzo, Daniel, and Beth said the acceleration was zero. Mary and Frank said it was constant and pointing downward. Cathy and John said it wasn’t zero, but it wasn’t constant either.

Which group do you agree with, or do you have a different opinion? Record your prediction and your reasons for it before continuing with this exercise.

#### *The Reasons*

Alonzo said the acceleration of the ball was zero because it was momentarily at rest at the highest point of the motion.

Frank said the acceleration was constant and downward because the acceleration of gravity is always constant and gravity was acting on the ball.

Beth said the acceleration was zero because it was negative when it was rising, since it was slowing down, and positive when it was falling, since it was speeding up, so it had to pass through zero at the apex of the motion.

John agreed with Beth that the acceleration was changing, but it was positive on the way up and negative on the way down because that was the direction the ball was moving. Therefore, it isn’t possible to measure the acceleration of the ball when it is at rest.

Which of these explanations are clearly wrong, and how do you know? Which one of the three explanations is correct?

#### *The Answer*

The acceleration of the ball is not zero at the apex of the motion. It is, in fact, downward and constant through the full motion of the ball, just as Frank said.

Alonzo said that the velocity was zero at the apex, therefore the acceleration was zero. What was wrong with this Alonzo’s reasoning?

Both Beth and John said the acceleration was not constant but changing in different ways. What was wrong with their reasoning?

Frank is right: the acceleration of gravity is downward and constant, but his answer refers to a theory that is known to be true. Can you provide a better explanation for the correct answer based on observation alone?

### *The Solution*

#### Key Principles

According to the Concept Review definitions (1a-c), velocity measures a change in displacement and acceleration measures a change in velocity of an object.

According to Principle 3c, positive and negative acceleration depend on both the changing speed and also the direction of motion.

#### The Facts of the Problem

The ball is rising upward and slowing down before the highest point. It is momentarily zero at the apex. It then moves faster and downward after it reaches the apex.

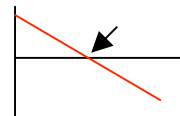
#### Applying Logic

The fact that the velocity is zero does not require that the acceleration is zero since velocity and acceleration do not measure the same thing. Therefore, Alonzo's reasoning is incorrect.

Beth and John disagreed on when the acceleration was positive and when it was negative. According to Principle 4c, moving upward and slowing down is a negative acceleration. But moving downward and speeding up is also a negative acceleration. Therefore, if the acceleration is negative before the apex and negative after the apex, it is probably negative at the apex.

Beth and John also assumed the acceleration was not constant but changing. Like Alonzo, they were probably confusing acceleration with velocity. But as this problem shows, any combination of positive, negative, or zero values of displacement, velocity and acceleration are possible with motion. The direction of the motion (John's assertion) does not by itself determine the direction of the acceleration. In the same way, increasing or decreasing speed by itself does not determine the direction of the acceleration. It is the combination of the two — both change in magnitude and change in direction — that determines the direction of the acceleration vector.

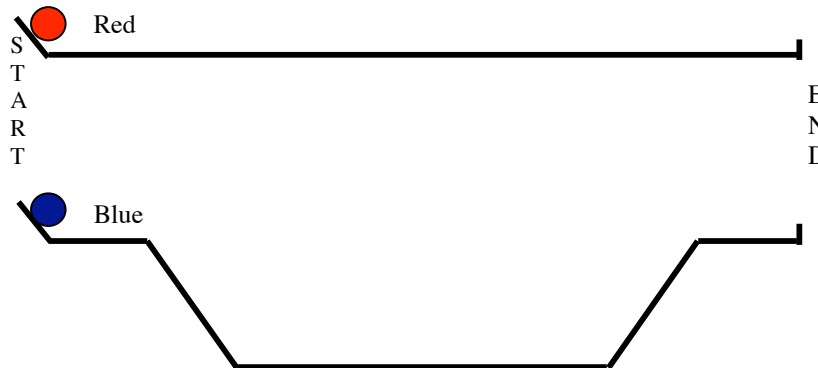
Observation alone doesn't prove that the acceleration is constant; measurements of the velocity at different times could be made to show that it is constant. Acceleration is also represented by the slope of a velocity vs. time graph. A graph of the ball's velocity vs. time — first positive, then zero, then negative — could also be used to convince a person that the acceleration of the ball was constant and negative (even when it was momentarily at rest) if the graph showed a straight line.



## 2. High Road, Low Road.

### *The Situation*

Mr. Nichols has brought in a special race track like the one shown in the figure. Two balls are released from the same height on a short ramp, so they travel with the same speed along the first horizontal section. The ball on the horizontal track, colored red, continues to travel with the same speed until it bumps into a block at the end of the ramp. The second ball, colored blue, follows a downward slope along a 'dip' and then rises up again to the original height before reaching the end of the ramp. "So," Mr. Nichols asks, "which ball wins the race?"



What is your prediction?

### *The Predictions*

Cathy says the red ball will win because the blue ball has to travel a farther distance.

John says the blue ball wins because it moves faster when it travels into the dip.

Frank agrees that the blue ball moves faster when it enters the dip, but it must also slow down by the same amount when it travels up the ramp again. Because these two accelerations cancel out, he believes the race will end in a tie.

Any comments? Are any of these arguments clearly wrong? On what basis do you make that determination?

### *The Answer*

Mr. Nichols releases both balls, and the blue one following the dip in the track is the clear winner. However, the class can also see that the blue ball did return to its original speed when it came out of the dip, so Frank was correct in his reasoning.

Can you explain why Cathy was wrong, even though the blue ball does travel a farther distance?

Can you explain how Frank could be correct in his reasoning, but still make the wrong prediction?

### 3. Galileo's Ramp

Mr. Nicholls explained to the class that Galileo wanted to study the properties of gravity but that the acceleration of gravity was too large. Therefore, he studied the effects of gravity on a 'gravity dilution device': a simple inclined plane. Mr. Nicholls wanted the class to reproduce Galileo's investigations with their own inclined planes.

Each pair of students was given a marble and a 1.5 meter ramp with a groove cut down the center. (You can do this experiment yourself on a board without a groove, or by placing two meter sticks next to each other with the marble rolling down the groove between them). One end of the ramp was raised up on a book; and an index card was taped to the other end of the ramp and to the table to make a smooth transition for the marble between the two surfaces.

The class was asked to start the marble from rest at the top of the ramp and to observe its motion traveling down the ramp and across the table. The class all agreed that the marble increased its speed rolling down the ramp, and after some discussion, that the ball rolled with a constant speed across the table top.

Next, Mr. Nicholls had the class start the ball from rest half-way up the ramp. Once again, all agreed that the ball moved slower across the table top than before. "So does that mean that the acceleration is less?", he asked.

About half the class thought that the acceleration was less because the marble was moving slower across the table. The other half thought the acceleration should be the same, but they weren't sure why.

After collecting position, time, and velocity data and calculating the acceleration, the class could see that in fact the acceleration on the ramp was constant.

What was wrong with the reasoning that a smaller velocity implies a smaller acceleration?

All things being equal, a smaller velocity would imply a smaller acceleration, according to the equation defining acceleration. But all things are not equal. What other factors have changed besides the velocity at the bottom of the ramp? Can you show using the equations how these changes cancel each other out, resulting in the same acceleration as before?

The class was then told to put a second book under the ramp and repeat the experiment with the marble starting from the top of the ramp. Everyone agreed that the velocity was greater across the table compared to the previous set-up. "So do you think the acceleration has changed, or not?" Mr. Nicholls asked. The whole class agreed that the acceleration of the marble was larger.

"How do you know?", the instructor asked.

"Because the marble is moving faster at the bottom of the ramp", Beth said.

"But a different speed at the bottom of the ramp didn't change the acceleration in the last experiment. Why should it imply a different acceleration now?"

How would you respond to Mr. Nicholls' question?

(In fact, after collecting more data, the class did prove that the acceleration of the ramp set at a greater angle was larger.)

## QUALITATIVE REASONING

### 1. Constant Acceleration?

A diver jumping off a diving board experiences an acceleration from the water in the pool that first causes her to slow down to a stop and then rise back to the surface of the water. One might assume that this acceleration is a constant acceleration.

How could you determine if the acceleration was constant or not?

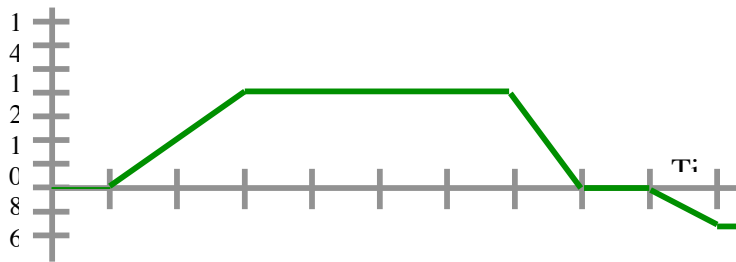
Assume you have the tools to directly measure position, time, and velocity of the diver in the water but not her acceleration. What data would you want to collect? How would you organize or plot the data to determine if the acceleration was constant? What would you look for specifically in the data or the plots that would prove that the acceleration was constant?

### 2. Velocity Graph

The figure below shows the velocity of a bicyclist traveling down a street.

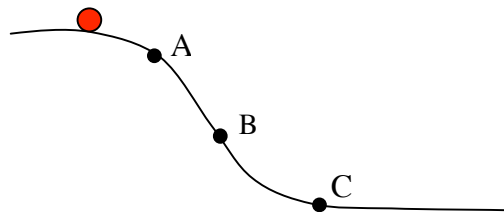
First, describe with words how the cyclist is moving in each region of the graph.

Second, sketch the graphs of position vs. time and acceleration vs. time for the cyclist.



### 3. Curved Ramp

Below is a picture of a mass sliding down a curved, frictionless ramp. Three points have been marked on the ramp, labeled A, B, and C



Compare the velocity of the mass at each of the three points. Is the velocity constant, continually increasing, continually decreasing, or varying in some other way as the mass slides down the ramp?

Compare the acceleration of the mass at each of the three points. Is the acceleration constant, continually increasing, continually decreasing, or varying in some other way as the mass slides down the ramp?

Be sure to give reasons for your answers based on the diagram and what you know about acceleration and velocity.

HINT: What did Galileo's ramp show about the relationship between acceleration and the angle of a ramp?

#### 4. The Brother's Race — Part 1

Jared challenged his brother Chris to a 50 meter race. Chris declined the challenge because Jared is three years older than him and definitely a faster runner. Jared agreed to even the odds by letting Chris start 10 meters ahead of him. At the starting signal the two brothers took off, with each running at a constant speed. The race ended in a tie.

Sketch a position vs. time graph showing the motions of both brothers on the same graph.

If Jared's constant speed was 5 m/s, what must have been Chris' speed? Can you answer this question by using logical reasoning and not by solving the equations for Chris' speed directly?

#### 5. The Brother's Race — Part 2

Dissatisfied with a tie, Jared challenges Chris to a second race. This time, instead of giving Chris a 10 meter lead, he is going to give him a 3 second head start.

Assuming Chris and Jared run with the same constant speeds as in the previous situation, who will win the race now? Can you figure out the answer without actually solving the equations for the time it takes each brother to reach the finish line?

Now sketch a position vs. time graph showing the motions of both brothers on the same graph.

#### 6. Motorcycle Cop

A motorcycle cop is hiding behind a billboard. A speeding car passes him, traveling at 20 m/s. The policeman begins chasing the speeder (starting from rest) at the same time that he passes the billboard, accelerating at a constant rate of  $2 \text{ m/s}^2$ .

Describe with words what you would observe if you were watching the motorcycle cop chase the policeman.

Draw a motion diagram of the chase.

Sketch position, velocity, and acceleration graphs of both vehicles. Plot the motion of both vehicles on the same graph.

Use the kinematics equations to determine at what time and at what position the motorcycle intercepts the car. Will the motorcycle have the same speed as the car at that point? How do you know?

### CAN YOU EXPLAIN THIS?

#### 1. Weight Drop

Try this yourself with two strings, each about 200 centimeters long, 12 hex nuts, and a wooden board.

Tie six of the hex nuts to the first string at equal distances, about 30 cm apart from each other, with the first one tied about 5 cm from the end of the string. For the second string, tie the nuts at 5 cm, 20 cm, 45 cm, 80 cm, 125 cm, and 180 cm from one end of the string.

Hold the first string so that the free end 5 cm from the first nut just touches the board. When you drop the first string, the clicks when each nut strikes the board will sound closer together. When you drop the second string of nuts, the 'clicks' will sound equally spaced.

Can you explain this?

Is there anything significant about the spacing of the hex nuts to create the equally-spaced sounds?

HINT: The weights fall due to gravity, and gravity is an example of constant acceleration. Examine the position vs. time equation, given in Concept Review 2e. How does 'when' relate to 'how far' according to that equation?

#### 2. Rounding a curve

A car turns a corner with a constant speed of 30 mph. Mr. Nichols claims that the car is experiencing an acceleration. Can you explain how this is possible?

#### 3. Walking Around a Table

Beth is not very happy with Mr. Nicholls. On her lab report he crossed out the word 'distance' and wrote the word 'displacement' above it. Beth challenged the teacher on it. "You are just being picky", she tells him, "distance and displacement mean the same thing."

“No they don’t,” he said, “Let me show you.” Mr. Nicholls asked Beth to stand next to the corner of the lab table. They agreed that the table was about 1.5 meters wide and 2 meters long. They also agreed that the corner of the table would be the origin.

“Now walk around the table, from one corner to the next, until you return to the corner where you started,” Mr. Nicholls told her. Beth did that. “Okay, now what distance did you travel?”, he asked her. “Seven meters”, she said. “And what was your displacement?”, he asked. “Seven meters,” she replied. “No,” he said, “Your displacement isn’t seven meters, it’s zero”

Can you explain why?

#### 4. The Velocity of a Toy Car

The class was completing an experiment where they were measuring the constant velocity of a toy car. According to Alonzo and Daniel, their wind-up car traveled 84 cm in 5 seconds, so they reported a value of 16.8 cm/sec for the velocity of the car. Mr. Nichol’s told them that their answer was not the velocity of the car.

Alonzo and Daniel did not make a mistake in their measurements or mathematics, and yet Mr. Nichol’s was still right. Can you explain this?

#### 5. Galileo’s ramp revisited

Frank and Daniel are exploring motion on Galileo’s ramp. The class has already demonstrated to their own satisfaction that the acceleration of a marble starting from rest on a ramp set at a certain angle is constant. Frank and John want to measure the acceleration of the marble when the initial velocity is not zero, so John gives it an initial velocity with a quick push right before Frank starts collecting data.

After analyzing the data, the pair of experimenters discover that the calculated acceleration of the car has not changed!

Can you explain this?

#### 6. Kinematics graphs

Mr. Nicholls asked the class to sketch kinematics graphs of the vertical motion of a child swinging on a swing. (He told them to ignore the horizontal motion). Up was taken to be the positive direction and down was the negative direction. He asked for graphs of one complete swing: from the farthest point behind the bar, to the farthest point in front of the bar, to the farthest point behind the bar again.

The following set of graphs were sketched by Mary. Only the velocity graph is correct; the position and acceleration graphs are not. Can you identify what is wrong with each of the other two?

Can you sketch the correct position and acceleration graphs, based on Mary's correct velocity graph?

