

IV. Energy and Momentum

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WARM UPS

1. Suppose you jumped straight up into the air. In terms of momentum and impulse concepts, why is it important to bend your knees when landing?
2. You have two blocks with one twice as massive as the other, but the heavier one is moving half as fast. Which one has the greater kinetic energy, or are the kinetic energies equal?
3. How is an elastic collision different from an inelastic one?
4. What does it mean to say that a quantity is conserved?
5. What is the work-energy theorem? What is the impulse-momentum theorem?
6. First use impulse-momentum principle, then use conservation of energy principles to explain why a nut falling from a tree is more likely to crack if it lands on a concrete driveway than it if lands on a patch of dirt.

CONFLICTING CONTENTIONS

1. Knocking Over a Block

“Here’s a simple question for you,” Mr. Nicholls started to say one day.

“No,” retorted John, “your questions are never simple. They only sound simple.”

“Well then, try to answer this one, anyway. I’ve got two identical wooden blocks standing on this table. Here are two pendulums, each constructed from a wooden rod with a ball attached to the end hanging in front of each one. One pendulum ball is made of rubber and the other is made of clay. The two balls also have the same mass.

“I’m going to release both pendulums from the same height at the same time. When they swing down, they will strike their respective blocks. I’ve already chosen the proper height so that at least one pendulum ball will knock over its block.

“So here’s the question: will the rubber pendulum knock over its block, will the clay pendulum knock over its block, or are both equally likely to knock over their blocks?”

“I’ll get started,” said Cathy. “I’ve got nothing to lose. Besides, this class has shown me that it doesn’t really matter what other people think of you, anyway — at least not when it comes to science. All that matters is understanding why things happen the way they do.”

“And, if I may add,” said Mr. Nicholls, “There’s nothing wrong with being wrong when you get started, as long as you know why you are right when you are done. But continue with your prediction.”

“Okay”, she said, “here I go. First I say to myself, ‘what’s the difference between the clay ball and the rubber ball?’ Answer: the clay ball sticks when it hits the block and the rubber ball bounces off. So does that make a difference when trying to knock over the block? I think it does. I predict the rubber pendulum will knock over the block.”

“And why, exactly, do you think that will happen? Remember, a hypothesis isn’t just a guess, it’s an educated guess. It’s a guess with a reason behind it.”

“All right,” Cathy added, “I think the rubber bullet will knock over the block because it bounces back. That means it has a bigger change in momentum compared to the clay ball. So it knocks over the block, and the clay ball doesn’t.”

“I’ll buy some of that, but I think the conclusion is wrong,” responded Alonzo. “We are agreed that both pendulums are going to strike their blocks with the same speed, right? And the clay ball is going to stick to the block, so it’s going to stop dead — or at least be close to that, because it will move forward just a little bit with the block.

“So let’s say the two balls are moving 3 m/s right before they strike the blocks, and they both have a mass of one kilogram. The clay ball comes to rest, so its change in momentum is 3, since three minus zero is three.

“But let’s say the rubber pendulum bounces back at a speed of 2 m/s. Then it’s change in momentum is only 1, since three minus two is one.”

“Okay, we have two conflicting theories before us, and both of them have some logic behind them. Any other ideas?”, asked the instructor.

Daniel raised his hand. “I’ve got one,” he said, “I think the whole momentum thing is going down the wrong track. What about the energy? Okay, we agree that both pendulums hit the blocks with the same speed. Since they have the same mass, they also hit them with the same kinetic energy. I think that the same kinetic energy means they are equally likely to knock over their block.”

Mr. Nicholls nodded. “Another logical argument based on the facts of the problem. So now we have predictions for all three possibilities: the rubber pendulum knocks over the block, the clay pendulum knocks over the block, and both pendulums knock over the blocks. Of course, all three hypotheses can’t be right at the same time.”

“And,” Beth interjected, “none of them may be right, because the reason for the right answer may be wrong.”

“True,” agreed Mr. Nicholls, “but I’m happy to say that one of our three guesses is absolutely right.”

So, can you figure out which student was correct — Cathy, Alonzo, or Daniel — and also identify the faulty logic in the other two predictions?

2. Conservation of Momentum in an Explosion

“These are called dynamics carts”, Mr. Nicholls explained. “They are really just two wooden blocks with low friction wheels. But one of them has a spring loaded rod in it, so when I press on this trigger, it can shoot out. It also has two different notches, so you can change the size of the spring force.”

The class played with the dynamics carts. In each case, they began with the carts at rest in the center of the table. They quickly came to the conclusion that if the masses of the two carts were the same, then the two cars moved away from each other after the explosion with the same speed. That happened whether the spring force was large or small. They also observed that if one car’s mass was twice the other, then it moved half as fast after the explosion. (Of course, the class recognized the effects of friction on the dynamics carts, so the data was approximate, not perfect.)

“I have a question for you,” Mr. Nicholls said, “Based on what we have observed, and ignoring the effects of friction, of course, is it possible to make the one car so massive that it remains at rest after the explosion?”

“No,” Alonzo replied, “that would conflict with the law of conservation of momentum. The two cars start with zero momentum before the explosion, so their momentum must add to zero after the explosion. Since the one car has some momentum moving forward, the other car must have

momentum moving the other direction. If the car was at perfect rest, it's momentum would be zero, so that's not allowed."

"Wait a minute," Frank said, "I'm not so sure about that. Suppose you set up the spring car against the wall of the classroom. After the explosion, wouldn't the wall still be at rest? Isn't the wall of the classroom the same thing as a very massive cart? How is momentum conserved in that case?"

"But the wall is attached to the floor which is attached to the building, and so on," Beth suggested. "They are all one piece. Our dynamics carts are supposed to be frictionless, so they aren't attached to anything."

"That may be true," said John, "But come to think of it, maybe Frank has a point. We like to say that everything is frictionless in the ideal case, but what exactly is the effect of friction, anyway? Why does it make a difference? Maybe it is just the way to connect the car to the earth, just as the wall is connected to the earth.

"What I'm starting to wonder is this: are these conservation laws, like conservation of momentum and conservation of energy, really universal laws, and we ignore friction and mass and stuff like that to make things easier to understand, or are these universal laws really the special cases that only work if friction and mass and stuff like that aren't around?"

"You know, John," the instructor remarked, "I think you just hit on a very important point. Either we are going to believe that the universe is a logical and reasonable place that always acts based on a relatively few, relatively simple principles like Newton's laws of motion and conservation of energy, or we are going to have to conclude that the universe is really just too complicated for anyone to understand, and it just does what it does when it decides to do it.

"So, for once I'm going to tell you what I believe. I think the universe is an understandable place, and that it does function on the principles of a few fundamental laws of nature. So I believe in conservation of momentum in all cases, including a cart exploding against a wall, and even with carts that have friction.

"Therefore, I agree with Alonzo. It is impossible to have a frictionless cart so massive that it will not move after the explosion. But now, we have to explain two new situations suggested by Frank while at the same time preserving conservation of momentum. First, what happens to the momentum of the cart when friction is present, so that a cart may not move after the collision? And second, how do we show that momentum is conserved when the cart explodes against a stationary wall?

"And to make life a little easier for you, I'll give you a hint. Beth was right on target; the wall is connected to the floor, which is connected to the building, et cetera."

Are you up to Mr. Nicholls' challenge?

3. Conservation of Energy in an Explosion

“I’ve got another problem, Mr. Nicholls,” Frank said, “and I’m almost afraid to say anything after the conservation of momentum thing we just did.”

“Let’s give it a try, anyway,” the instructor said, “I’ll place my bets on the laws of physics any time. What’s the problem?”

“Okay. Here’s the thing,” he continued. “I was just working out the kinetic energy of the two carts after the explosion, and it doesn’t seem to be conserved. If the two carts have the same mass, then their kinetic energies are the same, just like the momentum. But if one cart has twice the mass and moves with half the velocity of the other, so that momentum is conserved, then the kinetic energy of the heavier one is eight times smaller than the kinetic energy of the lighter one. It just seems to me that if momentum is conserved, then energy can’t be. How can I conserve momentum and energy at the same time in a case like that?”

“But kinetic energy doesn’t have to be conserved,” Beth suggested, “That’s only true for elastic collisions, and this one isn’t elastic.”

“Okay,” Frank replied, “But what other energies are in the carts? We don’t have potential energy, and in the ideal case we wouldn’t hear the cars and there would be no heat produced. So what other energies are in the carts besides kinetic energy?”

“Frank is right about that,” Mr. Nicholls agreed, “After the explosion, in the ideal case at least, the two cars only possess kinetic energy.”

Can you help Frank out of his predicament?

HINT: read Mr. Nicholls’ last sentence carefully. The key to the solution is contained in one of his phrases.

QUALITATIVE REASONING

1. Work and Energy in a Race

Imagine a race between two blocks, with one block having twice the mass of the other. Furthermore, suppose both blocks have an identical, constant force applied at all times during the race.



The lighter block wins the race, of course, because by Newton's second law ($F=ma$) it has the greater acceleration. But when the race ends as the lighter block crosses the finish line, do they possess the same kinetic energy? If they don't, then under what circumstances will the larger block have the same kinetic energy as the lighter block when the lighter one crossed the finish line?

Hint: Use the work-energy theorem when answering this question.

2. Impulse and Momentum in a Race

Once again, imagine a race between two blocks, with one block having twice the mass of the other. Both blocks have an identical, constant force applied at all times during the race, and the lighter block still wins the race.

Now compare the momenta of the two blocks when the race ends. If they don't have the same momentum when the lighter block crosses the finish line, then under what circumstances will the larger block have the same momentum as the lighter block when it did win the race?

Hint: Use the impulse-momentum theorem when answering this question.

3. The Lesser of Two Evils

A car is heading toward you in your lane, and a crash is inevitable. However, you can either hit the other car head on, or swerve into a side wall — but that collision will also be head-on. Which of these two bad choices would be best for you, in terms of energy and momentum principles, or are both choices equally bad?

4. Starting a Train Moving

A train engine with twenty loaded freight cars is unable to get enough traction to begin moving: the wheels just continue to slip. The foreman uncouples the freight cars from each other, then connects them again with three foot chains that hang loosely between the couplings. Now when the engine is able to get all twenty cars moving, even though it applies the same force to the freight cars as before.

Compare these two situations using energy and momentum concepts. Develop an argument to show how it was possible for the freight cars to accelerate in the second case but not in the first, even though the applied external force was the same in both cases.

5. Elastic Collisions

Could you hear a perfectly elastic collision? Why or why not?

6. Graphing Momentum and Energy

Consider again the behaviors of the two pendulums in the exercise, **Knocking Over a Block**. Draw two graphs, one charting the kinetic energies and the other the momenta, of the rubber pendulum and the clay pendulum from the time they released until a few seconds after the collisions.

CAN YOU EXPLAIN THIS?

1. Car Bumpers

The rubber bumpers on the bumper cars at an amusement park are designed so that the cars will bounce off each other, adding to the fun of the ride. The bumper of a car, on the other hand, is designed with a different purpose in mind. In fact, a ‘good’ car bumper is supposed to be crushed on impact — adding to your expense to replace it, but protecting you and the more expensive parts of your car. Can you explain why a car bumper is designed to crumple on impact?

2. Grandfather Clock

A mechanical grandfather clock keeps time by the pendulum swinging in the case below the clockworks. These clocks also have a counterweight, which slowly falls as the clock runs. When the counterweight reaches the bottom, it is reset by pulling down on the other side of the chain it is attached to. If the weight isn’t reset, then the clock will eventually stop running.

What is the purpose of the counterweight in a grandfather clock? In other words, explain why the clock will stop if the weight isn’t reset.

Hint: Use energy concepts and the principle of conservation of energy to answer this question.

3. Water Bomb

This is suppose to be a true story. Two college students, as a prank, dropped a water balloon from a dorm window ten floors up onto the windshield of a friend’s car. The windshield shattered. What happened?

4. Steel Bullet, Rubber Bullet

A rubber bullet and a steel bullet with the same mass and traveling at the same speed are shot at a target. The rubber bullet, we have learned, is more likely to knock over the target, but the steel bullet is more likely to damage the target. Use momentum or energy concepts to explain why.

5. Explosion in Motion

A block is moving forward with a constant velocity. An explosion inside the block causes it to split into two pieces. The front piece has two-thirds of the block's original mass; the back piece has one-third of the total mass. The more massive front piece continues to move forward, but with one-and-a-half times the original velocity. Use conservation of momentum to show that the back piece is not moving at all.

6. Conservation of Momentum?

If momentum is always conserved, what happens to the momentum of a ball of clay when it strikes the ground and comes to a dead stop? And where did it come from, anyway, while it was falling, since it wasn't colliding with anything during the fall?

