The Use of GPS in Introductory Mechanics Courses

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Introduction

The Global Positioning System is a revolutionary technology that is beginning to pervade every aspect of contemporary life. A commercially available low-end handheld GPS unit can locate any position on the planet with a RMS error of 6 m, i.e. within a circle of uncertainty of 6 m. There are techniques by which this error can be corrected in real time or by post processing to within a circle of uncertainty of less than 0.5 m. The sampling rate of a typical GPS unit is 1 second, during which it captures the latitude, longitude and altitude. Therefore, for a moving object, the (x,y,z) coordinates can be collected every one second to generate a time dependent 3-D position vector. If the mass of the moving object is known, then the entire kinematics, dynamics and energetics of the object can be calculated and analyzed.

The curricular and technology issues involved here are

1) Fundamentals of geo-centric coordinate systems, projections and datums
2) Fundamentals of the GPS system including how it works and the orbital characteristics of the constellation of GPS satellites
3) Issues involved with the user interfaces of commercially available GPS systems
4) Issues involve with the software and uploading of GPS data (x,y,z,t) to a PC, and getting it into a form such that it can be uploaded onto a spreadsheet type program
5) Programming the spreadsheet to extract the kinematic, dynamic and energy variation as a function of time
6) Analysis of the kinematics, dynamics and energetics of the moving object

It makes the learning of mechanics contextual to the students as they examine real life motion that they are actually involved in. GPS technology is rapidly evolving with the result that increasingly capable handheld and inexpensive GPS units are commercially available. This makes their use in the teaching and learning of mechanics increasingly viable. Video-capture technology allows the examination of 2D real life motion. GPS technology allows the examination of 3D real life motion. This allows real life motion, within the constraints of GPS data acquisition, to become a laboratory for exploring concepts of mechanics. It gives a powerful capability of connections ideas from the physics classroom to connect to the experiential domain of the student. It makes running, sailing, driving, biking, skydiving and other such outdoor activities a very rich arena for physics teaching/learning.

The Global Positioning System (GPS) and the Experiential Learning of Mechanics

The Global Positioning System (GPS) is a technology that allows a person with a GPS receiver to locate their position on Earth with both great accuracy and precision. The GPS technology was created by the U.S. Department of Defense, and now has entered the civilian domain with a wide array of applications. In essence, the GPS system consists of an array of 26 satellites, in which two of the satellites are spares. The constellation of 24 GPS satellites are in low Earth polar orbits spaced 15 degrees apart, so as to give coverage to the entire planet. From any given position on Earth, a GPS receiver can determine its position on the surface by having a line of sight to at least three GPS satellites, by means of triangulation. If more than four satellites are accessible to the receiver, the position can be determined with a greater accuracy.

Each GPS satellite broadcasts its ephemeris (orbital data). When a GPS unit is switched on, it downloads the ephemeris (orbital parameters and characteristics defining the position of the satellite as a function of time. This allows the receiver to measure the time taken by a signal from the satellite to the receiver. If more than GPS 3 satellites are in line of sight of the receiver, the geographic position of the receiver can be determined.

Upon making contact with at least 3 satellites, the GPS unit can locate its position on the Earth to within a circle of uncertainty of a couple of meters. The more the satellites it a direct line of sight with, the lesser the positional uncertainty, such that the position can be located within an root mean square (RMS) error of 1 m. A GPS unit captures the planimetric coordinates (Latitude/Longitude or the Easting/Northing) and the altitude, with a frequency of 1 hz. This means that for a moving object, the (x,y,z,t) coordinates are captured every one second, constructing the position vector of the moving object. This contains the complete description of the motion of the object. If the mass of the moving object is known, the net impelling forces and the energetics of the object can be unfolded as well.

GPS and Kinematics

Kinematics is the mathematical description of motion, its fundamental concepts are attributed to Galileo. Newton subsequently developed calculus to generalize Galileo's ideas such that motion in its general form could be described mathematically. All of kinematics of an object is known if the spatial time variation of a moving object is known. In essence, the GPS unit captures the position vector of an object moving on the surface of the Earth, every one second. The rate of change of this position gives the 3D velocity vector of the object. The rate of change of velocity is the 3D acceleration of the object. The time dependent position, velocity and acceleration contain the complete description of the motion of the object.

Due to the fixed sampling rate of one second and the requirement for the line of sight of at least three satellites there are some constraints in the type of motion that can be captured accurately by a GPS receiver. The sampling rate of 1 Hz does not allow the GPS unit to capture any changes in position that occur very rapidly within an interval of one second. The requirement for the GPS unit to lock on to at least three satellites makes motion under vegetative canopies or in the vicinity of very tall buildings not very viable for analysis.

GPS and Dynamics

Dynamics is the study of the causes of motion, and brings to light the concept of forces. If the mass and the time varying position vector of an object are known, the time variation of the net force vector acting on the object can be calculated. This is summarized by Newton's second law of motion, which states that the net force of an object is equal to its rate of change of momentum, which is also equal to the product of the mass and the acceleration.

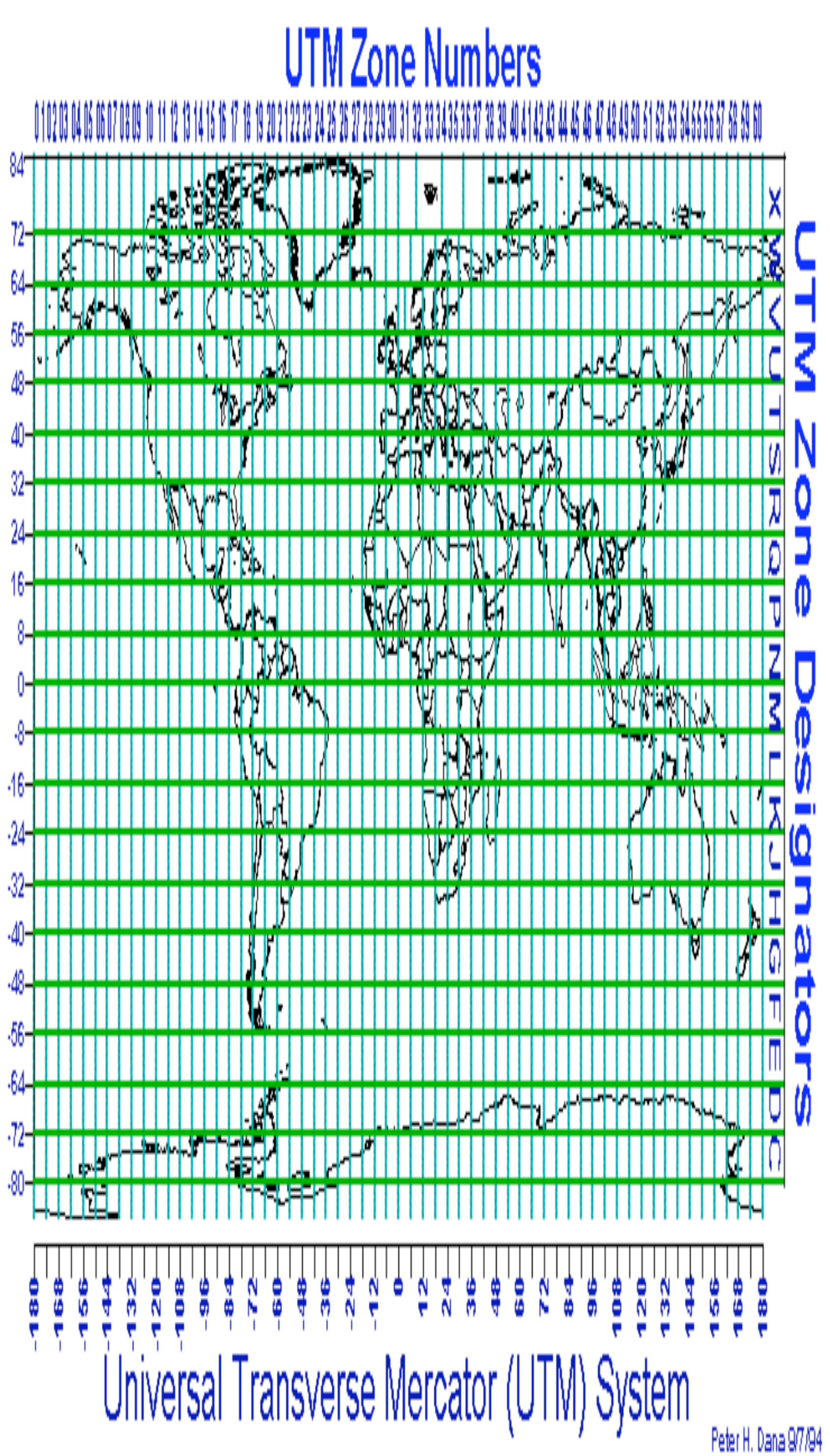
GPS and Energetics

Energetics is the study of the physical effort/work expended by forces on objects causing them to change position. Taking the Geoid (a gravitational equipotential surface) to be the location of zero altitude, the potential and kinetic energies of a moving object can be readily calculated from its time varying position vector and mass. The sum of potential and kinetic energies gives the total mechanical energy of the object. The change in the total mechanical energy is equal to the total work done by non-conservative forces, which typically in this case are external engines and the force of friction. Therefore, the knowledge of variation of total energy opens up many possibilities for the examination of the law of conservation of energy. The time variation of total energy allows the exploration of the concept of net power impelling the motion.

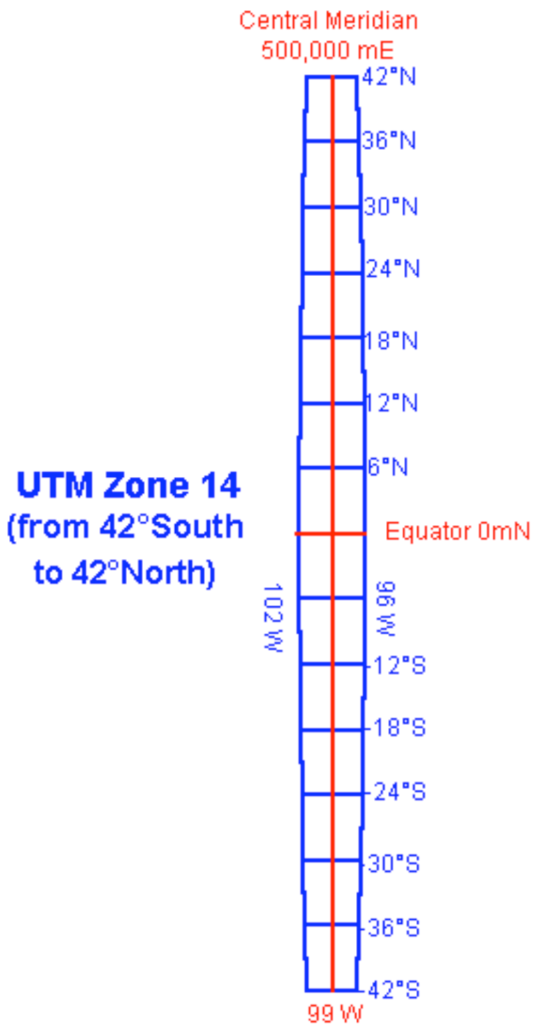
Geodata and the Universal Transverse Mercatur (UTM) Coordinate System

Geo-coordinate systems give the position of an object on the surface of the Earth. The Earth is not a perfect sphere and in reality is an undulating oblate spheroid that is also known as a Geoid. The planimetric position of a point on the surface of a spheroid can be represented by two coordinates which typically can be a latitude and longitude. The altitude is represented with respect to a gravitational equipotential surface, which is commensurate with sea level on the oceans. The exact position of this equipotential surface which represents zero altitude is more difficult to delineate exactly on land surfaces, and hence, the determination of altitude by a GPS unit has a greater uncertainity than of the planimetric coordinates.

A knowledge of the time varying position vector **R**(x,y,z,t) with the spatial and temporal coordinates in a rectangular grid and in MKS units would make it more amenable to analysis. To this end, it is the most useful to get the positional coordinates in the UTM coordinate system. The UTM coordinate system is depicts the Earth in rectangular grids and defines two dimensional, horizontal, positions. UTM zone numbers designate 6 degree longitudinal strips extending from 80 degrees South latitude to 84 degrees North latitude. UTM zone characters designate 8 degree zones extending north and south from the equator.



http://www.colorado.edu/geography/gcraft/notes/coordsys/gif/utmzones.gif



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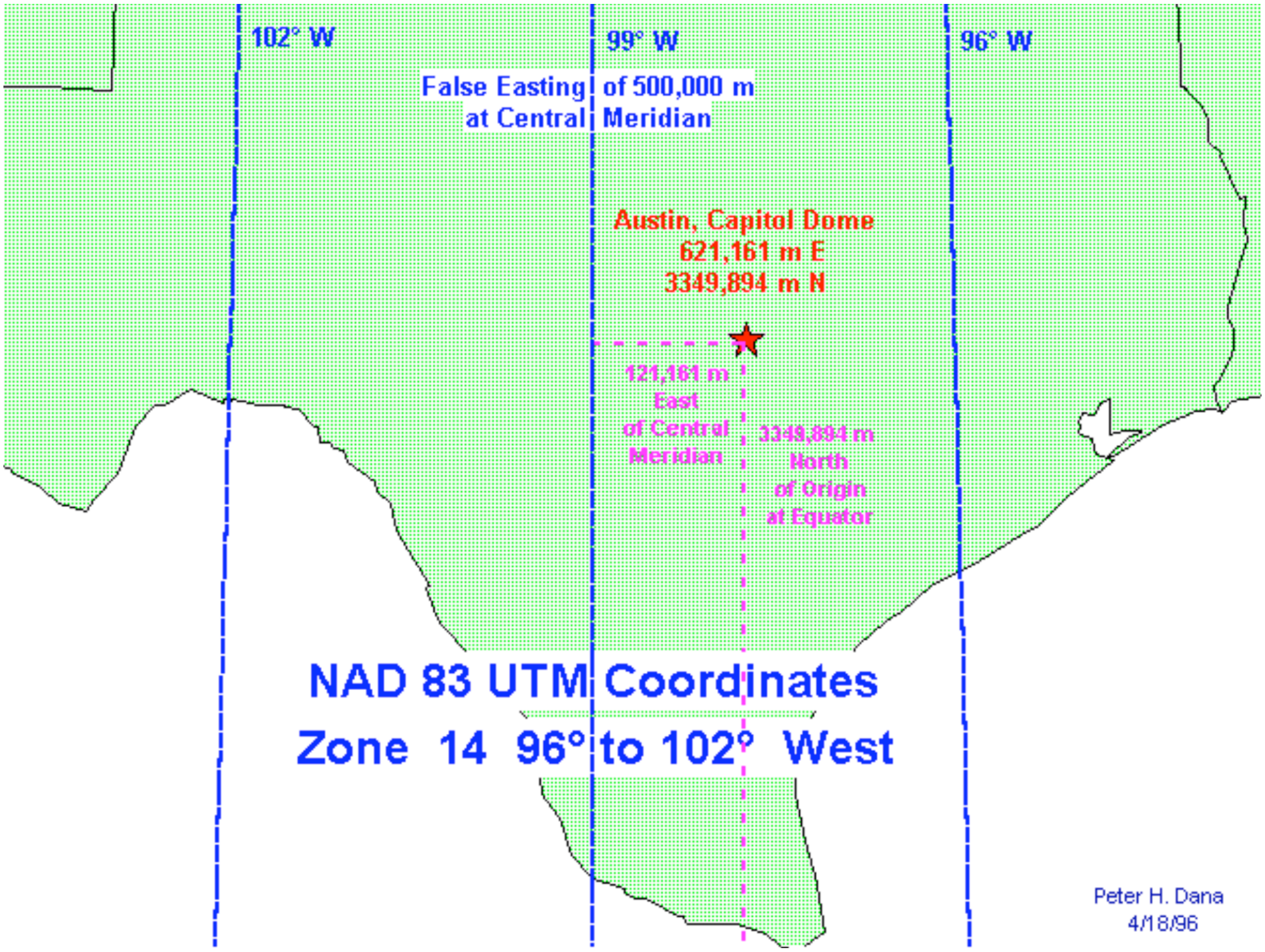

http://www.colorado.edu/geography/gcraft/notes/coordsys/gif/utm14det.gif

Eastings are measured from the central meridian (with a 500km false easting to insure positive coordinates).

Northings are measured from the equator (with a 10,000km false northing for positions south of the equator)

Procedure

1) Collecting Data from a GPS unit

There are many commercially available GPS units that are capable of collecting Positional data suitable for mathematical analysis .Garmin, Magellan and Trimble are some of well known brands, models of which are readily available at prices below $200 per unit, which are suitable for use in the physics classroom. The procedure described here pertains to a Garmin unit, but applies equally to most other handheld GPS units.

2) Downloading Data from a GPS unit to a PC

There are many handheld GPS units available that can be used for 3D position capture of a moving object, and then downloading the data to a PC for analysis. There are many shareware software packages available which facilitate the downloading of GPS data. One of the versatile shareware programs for downloading GPS data is the **GPS Trackmaker** which can be downloaded from the website

http://www.gpstm.com/

The following procedure must be followed to download the data from a GPS unit using the **GPS Trackmaker** software :

i) Run the **GPS Trackmaker** program on the computer. First of all you will set the coordinate system of the data to the UTM system as follows :
   - click on the 'Tools' button, and then click on 'Options'. This will open up a new window
   - click on the 'Coordinates' tab
   - click on the 'Rectangular Grids' radio button, and select 'UTM (Universal Transverse Mercator)' Now the data will be displayed in the UTM coordinate system, which consists of Zone , Easting and Northing. An altitude and time is also associated with each planimetric data point.
   - Also, click on the 'Datum' tab and note the Datum associated with the coordinate system
   - Click on OK

ii) Now we can download the data from the GPS unit to the PC using the **GPS Trackmaker** program; ***The data to be downloaded for analysis must be from the 'Active Log' only, and the 'Saved Log' in some instances removes the time stamps associated with each position***.

- connect the GPS unit using a data cable to the PC serial port (or any other available Comm port)
- click on the 'Interface' tab and then you can choose the brand of the particular GPS unit that you are using. In this case, we will be using the GARMIN unit, and so click on the 'Garmin' tab; this will open another window
- make sure that the proper 'comm port' has selected (i.e. the comm. Port into which the data cable is plugged into)
- click on 'product ID' button; this will identify the model of the the particular GPS unit and open up communications with the unit
- click on 'tracklogs' and this will download the position vs time data from the GPS unit
- Make a note of the number of points in the 'Tracks' field to ensure that viable data has been collected. The data is collected at a rate of one 3-D position per second.
- Click on 'Exit' to close the interface window.
- -Make sure that the coordinate system as displayed on the top right of the display is set to UTM (and not Latitude, Longitude). You can click on the tab and change to the UTM system if need be.
- You should now be able to see the planimetric data tracks on the UTM grid on the screen
- You may have to click on the tab next to 'Tracks and Routes' field to be able to zoom into the tracks of the motion to be analysed. ***Recall that the downloaded data must be from the 'Active Log' only as these positions come with an associated time stamp.***

- Click on the ‘File’ button on the top left corner and save the tracklog as a ‘GPS Trackmaker Text File’ with has a *.txt format, into a known location on your hard drive

3) Processing the Data on a Excel Spreadsheet to extract the position vector (x,y,z,t) in MKS units
    i) Navigate to the directory at which you have saved the data in a text file, and double click on the *.txt file. This will open up the file. Note the columns of numbers. Close this text file, as this was just to inspect the data.
    ii) Open up a blank Excel spreadsheet. Click on ‘File’ and then select ‘Open’, and point to the folder in which you have the data stored in a *.txt file. Make sure that ‘Files of Type’ field at the bottom of the ‘Open’ window is set to ‘All Files *.*’ type
    iii) Select the *.txt file and click on ‘Open’; this will bring up the ‘Text Import Wizard’ window. Select the ‘Delimited’ radio button and then click on ‘Next’
    iv) Click on the ‘Comma’ selection in the ‘Delimiters’ field and then click on ‘Next’
    v) Click on ‘Finish’; the data will now be displayed in columns on the spreadsheet.
    vi) Delete the non-column data headers which give information about the coordinate system, by clicking and dragging the cursor to select the field and then clicking on ‘Edit’ and then ‘Cut’
    vii) Note the data in the columns; you need to identify the Easting, Northing, Time and Altitude. The rest of the columns are not needed and can be deleted. You need to have an idea of the Geographic location of where the motion was executed, and what type of motion it was, such that you can have a basis for identifying the Easting, Northing and the Altitude. The needed columns can be identified as follows : Northing , Easting, Time, Altitude

viii) The Time will be in the format of Hours : Minutes : Seconds

You will need to use the HOUR(cell address), MINUTE(cell address), and SECOND(cell address), Excel Functions to create three more columns. Create a fourth column that converts the time from Hours, Minutes and Seconds to a column with Time in seconds only. Then click and drag down the Time columns, which will repeat this calculation for the entire time data, converting it all to seconds.

Now you should have the entire position vector in four columns on the spreadsheet as Easting (X axis in meters), Northing (Y axis in meters) and Altitude (Z axis in meters) and Time (in secs)

4) Uploading the Position Vector onto Vernier's Graphical Analysis Program :

Open the 'Graphical Analysis' program and make sure that you define the column's in Graphical Analysis as Easting, Northing, Altitude and Time. You do this as follows:

i) By default, there are two columns on Graphical Analysis labeled X and Y. Click on top of the first column 'X' and change the name to 'Easting' and the short name to 'X' and the units to meters.

Similarly, click on top of the 'Y' column and change the name to 'Northing' And the short name to 'Y' with units of meters.

ii) Now you need to add two more columns labeled Altitude and Time respectively. You do this by going to 'Data' and then 'New Manual Column'. Enter the name as 'Altitude', short name as 'Z' and units as meters. Repeat the process to create another manual column, with name as 'Time', short name as 'T' and units as seconds.

Now you can copy and paste the Easting (X), Northing (Y), Altitude (Z) and Time (T) columns from Excel to Graphical Analysis.

5) Unfolding the Kinematics, Dynamics and Energetics of the moving object, and graphical visualization of the results.

This data when uploaded into a spreadsheet or Vernier's 'Graphical Analysis' program and analyzed in the following manner:

a) The time derivatives of x(t) , y(t) and z(t) give v_x , v_y , and v_z ; the syntax for this in Graphical Analysis is derivative("x", "t") to give v_x , where x is the Easting column and t is the time column. You repeat a similar procedure to get v_y and v_z
b) The net speed can be calculated by the 3-D Pythagorean Theorem as v = $\sqrt{(v_x^2 + v_y^2 + v_z^2)}$; the syntax for this in Graphical Analysis is sqrt(v_x^2 + v_y^2 + v_z^2)
c) The magnitude of acceleration can be calculated as the time derivative of speed; the syntax for this in Graphical Analysis is derivative(**"v" , "t"**), where **v** and **t** are the Speed and Time columns respectively.
d) The net force on the moving object can be calculated by multiplying the acceleration column by the mass of the moving object
e) The net power delivered to the motion can be calculated as the product of net force column and the speed column
f) Similarly, the momentum (mv) , kinetic energy (1/2 mv^2) and potential energy (mgh) of the moving object can be calculated as the mass, speed and altitude of the moving object are known

All of the above quantities are available as columns of values in Graphical Analysis, as a function of time, and can be visually inspected as graphs.

6) Analysis of Results

This is perhaps the most interesting part of all. The velocity, force, net power, total energy and momentum graphs contain within them all the nuances of the subject of mechanics. The investigations and explorations into real-life motion comes with the possibilities of students making their own discoveries, living out the process of science and most importantly learning by doing.

Possible Investigations in Mechanics with GPS data

This capability brings in a unique dimension to the learning of introductory mechanics, whether it is at the high school or at the college level. It can help connect the concepts learned in the classroom to the actual experiential domain of the students. Students can take a GPS unit and take a car ride, run, bike, walk, sail, power-boat, fly in an airplane, skydive and capture the time varying position vector.

This opens up a wide array of possible investigations by the students. Some examples of the activities that can be done are:

1) Net power delivered to a moving object and the efficiency of the driving engine, may it be a car, boat or the human body.
2) Examination of the conservation of energy $\Delta(KE + PE) = \Sigma W_{NonConservativeForces}$
3) Observing how 'rate of change of momentum = net force'
4) Examination of uniform and non-uniform circular motion
5) Estimating the drag force on an accelerating car
6) Freely falling motion can be examined in a skydive
7) Estimating the energy efficiency of an automobile engine

Sample#1 GPS Activity Assessment Sheet (for any moving object):

Once the time dependent position vector (x,y,z,t) of a moving object of a known mass is captured, this data can be processed on Graphical Analysis as shown in the above procedure to calculate the speed, acceleration, net force, net power, kinetic energy, potential energy, total energy and momentum.

1) What are the maximum and minimum values of the speed and acceleration?

2) What are the maximum and minimum values of net force?

3) Does the most rapid change in the momentum vs time graph occur at the same time as the maximum net force ? Explain your observations.

4) Why does the net power delivered to the motion vary about zero? What is the physical interpretation of both negative and positive net power?

5) If the object is moving in a (near) circular path approximate the radius of This curve and find the speed of the object at this time. Calculate the centripetal force mv^2/r . How does this value compare with the net force at the time the object is moving in the (near) circular path ?

Sample#2 GPS Activity Assessment Sheet for a Moving Car:

Capture your 3-D position vs time with the car staring from rest and accelerates to a maximum constant velocity. Process this data as outlined in the procedure to get the kinetic energy, potential energy and net-power vs time graphs.

1) Calculate the Total Energy column by adding the Kinetic Energy and Total Energy Column. From this column, determine the change in total energy in the time interval elapsed during the acceleration of the car from rest to a constant maximum speed. This change in the total energy should be equal to the net work done on the car by all the forces acting on it (force of the engine and frictional drag). Dividing the net work by the time interval elapsed will give you the average net power delivered to the car during acceleration.

Compare this average net power delivered to the motion with the maximum horsepower rating of the car engine. How can you estimate the power dissipated due to the frictional drag due to air on the car? Please state any assumptions that you may make (i.e. that the average power delivered during acceleration is x % of the maximum power output of the engine)

2) Integrate the power vs time graph on Graphical Analysis for the time that the car was accelerating, and this will give you the net work done in accelerating the car. Compare this to the value obtained for the net work obtained in Part 1.

A gallon of gasoline contains about $132x10^6$ joules of energy, which is equivalent to 125,000 BTU or 36,650 watt-hours.
(http://science.howstuffworks.com/gasoline1.htm)

Based on the above information, can you calculate the amount of gasoline burned by the car ? Can you estimate the gas mileage i.e. miles per gallon of the car if it was to continue being impelled forward by the same average net power (as determined in part 1). Compare this to the known gas mileage of your car.

An Example of GPS Motion Data

Examples of GPS data acquired by Gainesville College Students are included with this writeup as both Vernier Graphical Analysis files.

1) Chris Gibbs Skydive – Spring Semester 2004 (Excel and Graphical Analysis files)
2) Michael Ginn's Boat Ride –Spring Semester 2004 (Graphical Analysis)